Getting Started with X-12-ARIMA Diagnostics — DRAFT

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After running X-12-ARIMA (or X-12 for short), you may find yourself asking, "How do I know if the program gave me a good seasonal adjustment?" This paper will explain the basics of the different diagnostics available in X-12-ARIMA and will give you some guidance on what options can help you get a better adjustment if the diagnostics point to a possible problem. We will not try to cover modeling diagnostics in this paper, though we will briefly discuss some of the options for regARIMA modeling.

For this document, we assume that you have already read "Getting Started with Win X-12, a Windows interface for X-12-ARIMA Version 0.3" (Hood, 2010), the documentation for Win X-12 (Lytras, 2009), or "Getting Started with X-12-ARIMA (Version 0.3), Using the Command Prompt on Your PC" (Hood and Monsell, 2010). We assume that you have a basic understanding of how to run X-12 and how to find the diagnostics information in the output file. This information is not essential to understanding the diagnostics, but may be helpful.

We also assume the reader has a general understanding of time series, ARIMA modeling, and the basic concepts of seasonal adjustment. For basic information on ARIMA time series modeling and basic seasonal adjustment, see Makridakis, Wheelwright, and Hyndman (1998). For detailed information on the X-11/X-12 method for seasonal adjustment, see Ladiray and Quenneville (2001). For information about new features in X-12-ARIMA, see Findley, Monsell, Bell, Otto and Chen (1998). For a more advanced treatment of ARIMA time series modeling and time series analysis, see Box, Jenkins, and Riensel (1994) or Brockwell and Davis (2002). For information about how some of the diagnostics have been used in the past, see Findley, Monsell, Bell, Otto and Chen (1998) and Findley and Hood (2000). You will also find more details of the seasonal adjustment diagnostics in the *X-12-ARIMA Reference Manual* (U.S. Census Bureau, 2009).

This paper does not cover the diagnostics for ARIMA modeling, advice on how to fix modeling problems, or diagnostics to help you decide whether or not to forecast the series.

The diagnostics output has changed very little in recent versions of X-12-ARIMA and doesn't vary between PC and Linux versions of the program.

This document is organized into the following topics:
1. Series that cannot or should not be seasonally adjusted
2. Identifying nonseasonal series — spectral graphs, M7 and the D8 F-test
3. Features of a quality adjustment
4. Residual seasonality
5. Identifying areas of instability — sliding spans and history
6. Possible causes of instability — extreme values and regression effects
7. Making changes to some of the more common options

1.   Series That Can Not or Should Not Be Seasonally Adjusted

Some series will not run through X-12. X-12 needs at least four years of data to run. Series with too many zero values in a row will also cause estimation problems for X-12 and can't be seasonally adjusted.

Series **should not** be seasonally adjusted when there is no clear evidence of seasonal behavior. If the original (i.e., unadjusted) series is not seasonal, then this series is already seasonally adjusted in that it does not contain a seasonal component.

Many official agencies will not publish seasonal adjustments if the stability diagnostics are unacceptable. This is more of a matter of preference and of the particular use of the seasonally adjusted series.

2.   Nonseasonal Series — Spectral Graphs, M7 and the D8 F-test

If we have a series with at least four years of data, the first step is to see if the series is seasonal. If the series is long enough (about 60 data points, or 5 years of monthly data), the best diagnostic to detect the presence of seasonality is the spectral graph. X-12 also has F-tests and an M7 diagnostic computed from the combination of several F-tests.

2.1   Graphs of the Original Series

The first step to identifying clear evidence of seasonal behavior is to graph the original series. Look for obvious seasonal patterns in the series, possible outliers, or zero values.

2.2   Spectral Graphs

You can analyze any time series from two different points of view: time and frequency. The two approaches are complementary, giving us different views of the same time series. To analyze a series in the frequency domain, we measure the strength of the different frequencies in decibels. The graph of the frequencies versus the decibels is called the *periodogram* or the *spectrum*, or as an adjective, *spectral graphs*. Spectral analysis allows us to see the relationships between the frequencies.

For monthly series with a significant seasonal component, the amplitudes that dominate are at frequencies of 1/12, 2/12, 3/12, 4/12, 5/12, and 6/12. If something happens regularly every year, then we see 1/12 of that cycle every month, and we will see a spike in the spectral plot at 1/12. Similarly, if something happens regularly every 6 months, then we see 1/6 of the cycle every month, and we will see a spike at 1/6 or 2/12. If something happens regularly every 3 months, then you will see a spike at 1/3 or 4/12, and so on.

For a quarterly series with a significant seasonal component, the amplitudes that dominate are amplitudes associated with components that repeat every year (i.e., four quarters) or every two quarters. So for seasonal series, we will see peaks at the frequencies associated with 1/4 cycle per quarter and at 2/4 cycle per quarter.

Also on the graph are frequencies for the *trading day* effect—the effect that comes from the weekday composition of the month. For example, if some retail sales series has very high sales on Friday and Saturday, and in a given month there were five Fridays and five Saturdays instead of four, then there would be increased sales that month because of the weekday composition of the month. The trading day frequencies were

identified by Cleveland and Devlin (1980). For monthly flow series, the most prominent trading day frequency is at 0.348. Another common frequency for series with trading day is at 0.432.

Before the spectrum is computed, we need a detrended series or the trend dominates the spectral graph and it is difficult to see anything else in the graph. For the original series, X-12 uses the differenced original series for the spectrum. Also, if there is a log transformation, X-12 will use the differenced, transformed series.

X-12 estimates several different spectral graphs. To look for seasonality in the original series, we look at the spectrum of the original series in Table G 0. If you are seasonally adjusting the series, and X-12 doesn't see any peaks in the original series, then you will get a warning from X-12 that the series should not be a candidate for seasonal adjustment. Conversely, if you are not asking for a seasonal adjustment, and X-12 flags seasonal peaks in the original series, then you will get a warning message that the series is seasonal and should be adjusted.

The rules for whether or not a peak is flagged as "visually significant" are somewhat complicated. First, the value at the seasonal and/or trading day frequencies must be six stars or asterisks higher than either neighboring frequency in the plot found in the output file of X-12-ARIMA. (To make the plot, the range of the frequencies is divided into 52 parts, so each asterisk in the plot equals 1/52nd of the range.) Second, besides the six-star rule, a seasonal or trading day peak also must be higher than the median of the frequencies in the graph in order for X-12-ARIMA to flag the frequency as a peak. For more information on how spectrum peaks are flagged, see Soukup (2000).

In practice, the higher the peak is above its neighbors, the more important the peak. Also, if the peak is the dominant peak in the graph, you may also have some cause for concern. Conversely, if you have a peak, even if it is six stars high, but it appears to be on the level of the noise in the spectrum, then it might be just noise.

Spectrum diagnostics work best with medium-length series. If we get too many observations, especially in economic time series, the series has the potential to change too much. If the seasonality is changing, we also don't get a good estimate of the spectrum at the various frequencies. By default, the spectrum is computed for eight years before the end of the span for monthly series and the entire series for quarterly series. This can be one reason why the spectrum diagnostics may disagree with the other diagnostics for the presence of seasonality in the original, unadjusted series.

Let's look at an example of a spectrum graph for a monthly series. On the next page is a spectral graph from X-12 of an original (unadjusted) series. The seasonal frequencies are marked with an "S" and the trading day frequencies with a "T". The left of the graph is the lower frequencies, so the first "S" marks 1/12, the second "S" marks 2/12, and so on. Notice that the original series shows signs of very strong seasonality. The peak at 1/12 is 11 asterisks taller than its taller nearest neighbor which makes it "visually significant." There are also seasonal peaks at the frequencies 2/12, 3/12, and 5/12 cycles per month. (There is not a peak at 4/12. That peak is shorter than its nearest neighbor. This, however, is not a cause for concern if we want to seasonally adjust this series because the series still shows strong seasonality.)

Example—Spectrum of the original series for a series with strong seasonality

```
G.0  10*LOG(SPECTRUM) of the differenced, transformed Original Data (Table A1 or
B1).  Spectrum estimated from 1995.Jan to 2002.Dec.
++++++++++I++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++I
    -5.81I                         *                                        I   -5.81
         I                         S                                        I
         I                         S                                        I
         I                         S                                        I
    -8.20I                         S                               *        I   -8.20
         I                         S         *                     S        I
         I                         S         S                     S        I
         I                         S         S                     S        I
   -10.58I                         S         S                     S        I  -10.58
         I                         S         S                     S        I
         I                         S         S                     S        I
         I                         S         S                     S        I
   -12.97I                         S         S                     S        I  -12.97
         I                         S         S                     S        I
         I                         S         S          *          S        I
         I             *           S         S          *          S        I
   -15.36I             S           S         S          *          S        I  -15.36
         I             S           S         S         **          S        I
         I             S           S         S        S*           S        I
         I             S           S         S        S*          S*        I
   -17.74I             S           S         S        S*          S*        I  -17.74
         I             S         *S          S        S**        *S*        I
         I             S         *S          S        S*T        *S*        I
         I*            S         *S          S        S*T        *S*        I
   -20.13I*            S         *S          S        S*T        *S*        I  -20.13
         I*            S         *S*         S        S*T        *S*        I
         I*           *S         *S*         S       *S*T*       *S*        I
         I*           *S         *S*         S       *S*T**      *S*        I
   -22.52I*           *S         *S*         S*      *S*T**      *S*        I  -22.52
         I*           *S         *S*         S*      *S*T***     *S**       I
         I*      *    *S         *S*         S*      *S*T***     *S*T       I
         I*      *    *S        **S*        *S*     **S*T***    **S*T       I
   -24.90I**    **   *S*        **S*        *S*     **S*T****  **S*T       I  -24.90
         I**    *****S*        **S*        *S*    ***S*T****  **S*T       I
         I**    *****S*        **S*        *S*    ***S*T*******S*T       I
         I**    *****S*        **S**       *S*   ****S*T*******S*T       I
   -27.29I**    *****S*        **S**       *S**  *****S*T*******S*T       I  -27.29
         I**   ******S*       ***S**      *S** ******S*T*******S*T*      I
         I***  ******S*       ***S**     *S*********S*T*******S*T*    *  I
         I*** ******S**       ***S**    **S*********S*T*******S*T*    *  I
   -29.67I***********S**       ***S***   **S*********S*T*******S*T*   *  I  -29.67
         I***********S**      ****S***   **S*********S*T*******S*T*  **  I
         I***********S***     ****S****  ***S*********S*T*******S*T*  **  I
         I***********S*** *****S*******S*********S*T*******S*T** **  I
   -32.06I***********S******S*********S*********S*T*******S*T*****  I  -32.06
         I***********S******S*********S*********S*T*******S*T******  I
         I***********S******S*********S*********S*T*******S*T******  I
         I***********S******S*********S*********S*T*******S*T******  I
   -34.45I***********S******S*********S*********S*T*******S*T******  I  -34.45
         I***********S******S*********S*********S*T*******S*T******  I
         I***********S******S*********S*********S*T*******S*T******  I
         I***********S******S*********S*********S*T*******S*T******* I
   -36.83I***********S******S*********S*********S*T*******S*T********I  -36.83
++++++++++I++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++I
           S=SEASONAL FREQUENCIES, T=TRADING DAY FREQUENCIES
```

Example—Spectrum of the original series for a series with no seasonality
Note: The only peak at a seasonal frequency is at 5/12, but it is not visually significant because it is only three asterisks taller than its nearest neighbors.

```
G.0  10*LOG(SPECTRUM) of the differenced, transformed Original Data (Table A1 or
B1).  Spectrum estimated from 1995.Jan to 2002.Dec.
++++++++++I+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++I
    -7.05I                                                        *   I      -7.05
         I                                                        *   I
         I                                                        *   I
         I                                             *          *   I
    -9.07I                                        *    *          *   I      -9.07
         I                                        *    *      *   *   I
         I                                        *    *      S   *   I
         I                                        *    *      S   *   I
   -11.09I                                        *    *      S**  ** I      -11.09
         I                                 *      *    *      S*T  ** I
         I                                 *      *    *   * *S*T  ** I
         I                                 *     **   **  ***S*T  *** I
   -13.11I                                 *     **  *** ***S*T* ****I      -13.11
         I                       *    **   **  T** ****S*T* ***SI
         I                       *    S*   *** T** ****S*T* ***SI
         I                       *    S*  **S T*******S*T* ***SI
   -15.13I             *    *    S* ** **S T*******S*T** ***SI      -15.13
         I             *    *    S* ** **S*T*******S*T** ***SI
         I            **    *    S* ** **S*T*******S*T*******SI
         I            **    *    S** ** **S*T*******S*T*******SI
   -17.15I            **   ** S***** ***S*T*******S*T*******SI      -17.15
         I            **  *** S*********S*T*******S*T*******SI
         I           *** *** S*********S*T*******S*T*******SI
         I           ***S *** S*********S*T*******S*T*******SI
   -19.17I           ***S *** S*********S*T*******S*T*******SI      -19.17
         I        *******S *** *S*********S*T*******S*T*******SI
         I        ********S **** *S*********S*T*******S*T*******SI
         I     ** ********S* ***** *S*********S*T*******S*T*******SI
   -21.18I     ** ********S******* *S*********S*T*******S*T*******SI      -21.18
         I     ** *********S**********S*********S*T*******S*T*******SI
         I   *  ** *********S**********S*********S*T*******S*T*******SI
         I  ** ** *********S**********S*********S*T*******S*T*******SI
   -23.20I  ** ***********S**********S*********S*T*******S*T*******SI      -23.20
         I ** ****S*********S**********S*********S*T*******S*T*******SI
         I ** ****S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
   -25.22I *******S*********S**********S*********S*T*******S*T*******SI      -25.22
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
   -27.24I *******S*********S**********S*********S*T*******S*T*******SI      -27.24
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
   -29.26I *******S*********S**********S*********S*T*******S*T*******SI      -29.26
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
   -31.28I *******S*********S**********S*********S*T*******S*T*******SI      -31.28
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
         I *******S*********S**********S*********S*T*******S*T*******SI
   -33.30I*******S*********S**********S*********S*T*******S*T*******SI      -33.30
++++++++++I+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++I
         S=SEASONAL FREQUENCIES, T=TRADING DAY FREQUENCIES
```

2.3 D8 F-Tests and M7

Included after Table D8 in the X-12-ARIMA output are some ANOVA tables that help measure stable and moving seasonality. Generally for F-tests, one would use a cut-off value of 4.0, but because we have generally correlated data in the time series, a cut-off value of 7.0 is recommended by the authors of X-11-ARIMA (Lothian and Morry, 1978). The first ANOVA table given tests for the "presence of seasonality assuming stability" using the variation between months. The F-value from this table is referred to in Table F2.I as the "F-test for stable seasonality from Table D 8" and so has come to be known as the "D8 F-test". It is also sometimes referenced as FS, the F-value to test for seasonality. We want the F-value for the test assuming stability to be greater than 7.0. A second ANOVA table after Table D 8 labeled as the "moving seasonality test." It is sometimes references as FM, the F-value to test for moving seasonality. We want the F-value for the Moving Seasonality Test to be small.

Example—Table D8.A output from a seasonal series

```
D 8.A  F-tests for seasonality
Test for the presence of seasonality assuming stability.
                 Sum of       Dgrs of        Mean
                 Squares      Freedom        Square      F-Value
   Between months   8060.1099      11      732.73726    119.325**
        Residual     804.4320     131        6.14070
           Total    8864.5419     142

         **Seasonality present at the 0.1 per cent level.

  Moving Seasonality Test
                  Sum of       Dgrs of        Mean
                  Squares      Freedom        Square      F-value
   Between Years     26.5628      10        2.656283      0.522
        Error       560.1970     110        5.092700

         No evidence of moving seasonality at the five percent level.
  COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY
            IDENTIFIABLE SEASONALITY PRESENT
```

Also in the X-12 output, in Table F3, are 11 M statistics developed to point out characteristics of the series that may cause problems with the adjustment. One of these statistics, M7, was designed by the authors (Lothian and Morry, 1978) to determine whether seasonality can or cannot be identified by X-11/X-12. M7 compares the value of FS to 7.0 and also to the value of 3 times FM, meaning that the F-value for stable seasonality should be greater than 7.0 and greater than 3 times the value for FM. Its interpretation is straightforward: a value greater than 1.0 indicates no identifiable seasonality. We will discuss the other M statistics later in this paper.

3. Features of a Quality Adjustment

A seasonally adjusted series should not have any estimable seasonal effects. The presence of seasonal and calendar effects in either the seasonally adjusted series or irregular component is referred to as *residual seasonality*. Also, the level of the seasonally adjusted series should not have *bias*, meaning that the local levels of the original series and the seasonally adjusted series should be similar.

Some of the most important diagnostics in X-12 are the diagnostics to detect residual seasonality. X-12 has procedures that give us unbiased estimates of the seasonal component, so there are no diagnostics present to check for bias.

Other features are desirable, though not mutually exclusive. Many prefer adjustments that are stable, meaning that as new data are added and incorporated into the estimation procedure, the revisions to the past estimates are small. Big revisions can indicate that the initial estimates are misleading or even meaningless. Another useful feature may be smoothness, particularly if you are looking for turning points in the series. Unfortunately, a smoother series often has larger revisions, though this is not always the case.

4. Residual Seasonality

As when looking for seasonality in the original series, if the series is long enough, the best diagnostic to detect the presence of seasonality in the seasonally adjustment is the spectral graph. Besides the spectrum of the original series, X-12 computes spectral plots for the seasonally adjusted series (Table G1) and the irregular component (Table G2). On the next page is the spectral graph for the seasonally adjusted series that corresponds to the spectrum of the first series shown. Notice that after seasonal adjustment, the seasonal frequencies have all been suppressed. Because residual seasonality is an important problem, X-12 sends warnings about spectral peaks in the seasonally adjusted series and the irregular to the screen and the error file. The rule for identifying visually significant peaks is the same as for the original series.

You may notice in the heading of the graph that the spectrum is for the seasonally adjusted series has been detrended and modified for extreme values. This is also true for the irregular spectrum. (Therefore, the spectral graphs are computed from Tables E2 and E3.)

Residual seasonality in the seasonally adjusted series or the irregular can often be eliminated by either shortening the span of the data used for modeling or for the adjustment, changing the seasonal filter lengths, or both.

Other Tests that Should Not be Used

It is a common misconception that the M7 and D8 F-test available in X-12 are for the detection of residual seasonality. Both diagnostics were meant to help the user decide if the original series is seasonal, and some users, therefore, run the seasonally adjusted series back through X-12-ARIMA to look at the values for M7 and the D8 F. However, the M7 diagnostic was designed to indicate either a lack of seasonality or too much moving seasonality relative to the stable seasonality. The diagnostic is scaled based on empirical results on seasonal series at Statistics Canada, not to detect residual seasonality in already adjusted series. The M7 is susceptible to the model used and to the type of adjustment used by X-12-ARIMA. The D8 F is somewhat more stable than the M7 diagnostics, but is still dependent on the extreme values chosen in the X11 procedure. Some users will take the adjusted series and look at a significance test on the individual months or quarters to determine if there is significant residual seasonality. This diagnostic is also very susceptible to changes in the model. Running a seasonally adjusted series back through X-12 also requires modeling the already adjusted series. For more information, see "Assessment of Diagnostics for the Presence of Seasonality," Hood (2007).

Example—Spectrum of the seasonally adjusted series (corresponding to the series in Example 1)

```
G.1  10*LOG(SPECTRUM) of the differenced, transformed seasonally adjusted data
(Table E2).  Spectrum estimated from 1995.Jan to 2002.Dec.
+++++++++I+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++I
   -24.35I*                                                                 I    -24.35
         I*                                                                 I
         I*                                                                 I
         I*                                                                 I
   -25.99I*                                                                 I    -25.99
         I**                                                                I
         I**                                                  *             I
         I**                                       *          *             I
   -27.63I**                                       *          *             I    -27.63
         I**                                       *          *         *   I
         I**                                       *          *   *     *   I
         I**                                  *    *          ** *     *   I
   -29.27I***   **                            *    *          ** *     *   I    -29.27
         I***   **                            *    *          ** **        *   I
         I***   **                            *    *          ** **        *   I
         I***   **                            *    *          ** **       **   I
   -30.91I***   **        *                   *    *          ** **       **   I    -30.91
         I*******        **                   *    *          ** **    *  **   I
         I*******        **                   *    *          *****    * ***   I
         I*******        **                   *    *    **    *****    *****   I
   -32.55I*******        **                   *    *    **    *****    *****   I    -32.55
         I*******        **             ** *      ** **    *****    *****   I
         I********       **             ** **     ** **    ******   *****   I
         I********       **             *****    *** **    ******   *****   I
   -34.19I********       **    *         *****    *** **    ******    ******  I    -34.19
         I********       **    *         *****    *** **    ******    ******  I
         I********       **    *         *****    *** **    ******    ******  I
         I********       **    *         *****    *** **    ******    ******* I
   -35.83I********       **   **         *****    *** ***    ******    ******* I    -35.83
         I********       **   **         *****    *******    ********   ******* I
         I*********      **   **         *****    *******    T*******   ******* I
         I*********     ***   **         *****    *******    T*******   T********I
   -37.47I*********     **** **         *****    *******    T*********T*******SI    -37.47
         I*********     **** **         *****    ********   T*******S*T*******SI
         I*********     **** **         *****    ********   T*******S*T*******SI
         I**********  ******         *****    *********   T*******S*T*******SI
   -39.11I******************         ******    *********   T*******S*T*******SI    -39.11
         I**********S*******         ******    ********* *T*******S*T*******SI
         I**********S*******         ******    ***********T*******S*T*******SI
         I**********S*******         *******    *********S*T*******S*T*******SI
   -40.75I**********S*******         *******  **********S*T*******S*T*******SI    -40.75
         I**********S********         ********S*********S*T*******S*T*******SI
         I**********S********         ********S*********S*T*******S*T*******SI
         I**********S********         ********S*********S*T*******S*T*******SI
   -42.40I**********S********         ********S*********S*T*******S*T*******SI    -42.40
         I**********S********         ********S*********S*T*******S*T*******SI
         I**********S********         ********S*********S*T*******S*T*******SI
         I**********S********         ********S*********S*T*******S*T*******SI
   -44.04I**********S********         ********S*********S*T*******S*T*******SI    -44.04
         I**********S********        *********S*********S*T*******S*T*******SI
         I**********S********* ********S*********S*T*******S*T*******SI
         I**********S********* ********S*********S*T*******S*T*******SI
   -45.68I**********S*************************S*********S*T*******S*T*******SI    -45.68
+++++++++I+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++I
            S=SEASONAL FREQUENCIES, T=TRADING DAY FREQUENCIES
```

5.   Identifying Areas of Instability — Sliding Spans and History

Large revisions and instability may indicate that the seasonal adjustment is not useful and that X-12-ARIMA can not accurately identify any seasonal pattern that could be present in the series. If your series is long enough, you can run the series with the **slidingspans** spec and the **history** spec. For most series, you will need at least ten years of data. Details on the diagnostics and the data needed are given below.

5.1   Sliding Spans

Sliding spans detect instability in the seasonally adjusted series. Sliding spans compare different estimates from the seasonal adjustment from overlapping subspans of the time series data. You can look at comparisons between the seasonal factors or the seasonally adjusted series and the month-to-month (or quarter-to-quarter) changes in the seasonally adjusted series. The length of the span depends on the length of the seasonal filter and the date of the last point. For example, a 3x5 seasonal filter needs seven years of data, so X-12, by default, uses at least eight years (96 points for a monthly series). For 3x3 filters, X-12 uses spans that are six years long. For 3x9 filters, X-12 uses spans that are 11 years long. The spans start in January by default. So if the series ends in February, then there will be an extra two points at the end of the series past the default number of years. The last span ends at the last data point. For the second to last span, one year of data is taken off the end of the series, but the length of the spans stays the same, so the starting point for this span is a year earlier than the date for the last span. The other two spans are calculated similarly by starting the span one year earlier and keeping the length of the span the same. If possible, X-12-ARIMA looks at four spans. If there is not enough data for four spans, the program will calculate only three or two spans.

For most series X-12 will use 3x5 seasonal filters, which means you need 11 years of data to get four spans for the sliding spans diagnostics. For 3x9 filters, you will need more than 14 years of data to get four spans.

X-12 calculates a seasonal adjustment separately for every span using only the data in the span. This gives us possibly four different estimates of a large number of points. Of course, some of the points only have two estimates, and some only have one estimate. However, for the points where a comparison can be made, we can look at the estimates across spans. X-12 looks at the maximum difference between the estimates. For the seasonal factors, the seasonally adjusted series, the month-to-month (or quarter-to-quarter) percent change in the seasonally adjusted series, and the year-to-year percent change in the seasonally adjusted series, X-12-ARIMA looks for a maximum change between the spans that is greater than three, by default. X-12 then computes how many of the comparisons were greater than the cut-off value (again, three percent by default for most of the estimates). X-12 tells you how many of the points (and what percentage of the points) were above the cut-off.

In the example output below, the series goes from January 1989 to December 2002. The series uses 3x5 seasonal filters, so the span length chosen by X-12 is 96 points. The first span runs from January 1992 to December 1999, the second from January 1993 to December 2000, the third from January 1994 to December 2001, and the fourth from January 1994 to December 2002. Since we only have one estimate for the years 1992 and 2002, we can only make comparisons from January 1993 to December 2001. That gives us 108 points for the comparisons of the seasonal factors or seasonally adjusted series, and only 107 points for the comparisons of the month-to-month changes. For this series, none of the points had maximum percent differences greater than 3.0% in the seasonal factors. Only three months had a maximum percent difference in the month-to-month changes greater than 3.0%.

Example—Sliding spans output
```
S 2.  Percentage of months flagged as unstable.

  Seasonal Factors                                 0 out of 108 (  0.0 %)
  Month-to-Month Changes in SA Series              3 out of 107 (  2.8 %)

An adjustment is considered acceptable if the
  Percentage of unstable seasonal factors is less than 15%,
  Percentage of unstable seasonal adjustment values is less than 15%, and
  Percentage of unstable month-to-month changes is less than 35%.

An adjustment is considered unacceptable if the
  Percentage of unstable seasonal factors is greater than 25%,
  Percentage of unstable seasonal adjustment values is greater than 25%, and
  Percentage of unstable month-to-month changes is greater than 40%.
```

If you run the sliding spans diagnostic with four spans, and the diagnostics pass, then there is no evidence that the series is too unstable for seasonal adjustment. If the percent flagged are too high, it could be an indication your series has a lot of irregular. Below is an example from another series.

Example—Sliding spans output for another series
```
S 2.  Percentage of months flagged as unstable.

  Seasonal Factors                                45 out of 108 ( 41.7 %)
  Month-to-Month Changes in SA Series             48 out of 107 ( 44.9 %)

Summary Statistics for the Maximum Percent Differences across spans

     Minimum              :        0.06
     25th Percentile      :        1.36
     Median               :        2.44
     75th Percentile      :        4.10
  ->85th Percentile       :        4.78<-
     Maximum              :        9.93

     Standard Deviation   :        3.62

Histogram of the Maximum Percent Differences across spans

 Percent
 Differences      Frequency
     0.45 +################
          |####################
     2.26 +####################
          |#############
     4.07 +############
          |##########
     5.88 +#######
          |##
     7.68 +#
          |#
     8.59 +#

  One '#'= 1 observation[s]
  Breakdown of the maximum percentage differences of the
  Seasonal Factors for flagged months.
          Greater than or equal to  3.0% but less than  4.0% :  17
          Greater than or equal to  4.0% but less than  5.0% :  12
          Greater than or equal to  5.0% but less than  6.0% :  10
          Greater than or equal to  6.0%                     :   6
```

Notice that the 85th percentile of the maximum percent differences is 4.78. This means that if we raise the cutoff value from 3% to something larger than 4.78%, the sliding spans diagnostics would pass. We can raise the cut-off value to 5% with the argument **cutseas=5** in the **slidingspans** spec and run the series again. Then, if the sliding spans diagnostics passes with four spans, there is no evidence that the series is too unstable.

Example—Sliding spans output with the cut-off raised to five
```
S 2.   Percentage of months flagged as unstable.

  Seasonal Factors                                16 out of 108 ( 14.8 %)
  Month-to-Month Changes in SA Series             19 out of 107 ( 17.8 %)
```

There is additional output that is also useful. X-12 prints breakdowns of the average maximum percent difference by month and by year. This allows us to see if some months or some years are more unstable than others. We can also look at diagnostics for stable and moving seasonality and the M7 diagnostic to see if some spans are not seasonal.

Example—Additional sliding spans output
```
S 3.a  Breakdowns of unstable Seasonal Factors and Average Maximum Percent
       Differences across spans for Seasonal Factors
              January   :   3      (AMPD =   2.9)
              February  :   5      (AMPD =   3.9)
              March     :   5      (AMPD =   2.7)
              April     :   4      (AMPD =   3.8)
              May       :   0      (AMPD =   1.1)
              June      :   0      (AMPD =   1.7)
              July      :   6      (AMPD =   3.4)
              August    :   5      (AMPD =   2.6)
              September :   6      (AMPD =   3.6)
              October   :   3      (AMPD =   3.1)
              November  :   6      (AMPD =   3.7)
              December  :   2      (AMPD =   2.2)

              1993      :   0      (AMPD =   1.3)
              1994      :   5      (AMPD =   3.2)
              1995      :   8      (AMPD =   4.0)
              1996      :   8      (AMPD =   3.4)
              1997      :   8      (AMPD =   3.3)
              1998      :   7      (AMPD =   3.6)
              1999      :   7      (AMPD =   4.5)
              2000      :   2      (AMPD =   1.5)
              2001      :   0      (AMPD =   1.1)


Summary of tests for stable and moving seasonality from table D8 for each span
                                Span 1      Span 2      Span 3      Span 4
       Stable seasonality       117.41      124.10      141.73      151.77
       Moving seasonality         0.36        0.35        0.26        0.19
       m7                         0.19        0.18        0.17        0.16
       Identifiable seasonality?   yes         yes         yes         yes
```

5.2   Revision Histories

Revision histories give information about stability in the seasonally adjusted series or the trend. In history analysis, X-12 performs a sequence of modeling and/or seasonal adjustment runs on a sequence of increasing data spans, adding one point at a time to the end of the series. Available options in history spec of X-12-ARIMA include:
- Revisions for seasonal adjustment, seasonal factors, and trend,
- Month-to-month (or quarter-to-quarter) revisions for seasonal adjustment and trend,
- Revisions from projected factors to final factors, and
- Out-of-sample forecast errors and AICs for the regARIMA model.

To be able to use the revision diagnostics, you may need as few as seven years, but as many as 15 years of data depending on the seasonal filters. For most series, X-12 will choose a 3x5 filter (which uses seven years of data), so you will need about 10 years to get useful revision diagnostics.

As with the sliding spans, X-12 prints breakdowns of the revisions by month and by year. This allows us to see if some months or some years are more unstable than others.

6.   Possible Causes of Instability

6.1   Changes in the Seasonal Filter Lengths, Transformation, etc.

If you do not specify exactly what options you want X-12 to use, the options could change from run to run. For this reason, it is best to specify the transformation in the **transform** spec, any trading day or moving holiday effects in the **regression** spec, any outliers in the **regression** spec, the ARIMA model in the **arima** spec, and seasonal filter length in the **x11** spec.

6.2   Extreme Values

X-12-ARIMA has two different procedures to protect against extreme values. The worst outliers are prior-adjusted out of the series using regARIMA models. There is also an extreme value procedure inside of the X-11 module. We usually use the word *outliers* refers to outliers in the regARIMA model and *extreme values* for outliers identified in the X-11 algorithms.

To estimate the presence of extreme values, we first estimate a standard error from the irregular and then eliminate the influence of points with large irregular compared to the standard error. By default, points with irregulars
- Less than 1.5 times the standard error are not replaced and are given a weight of 100,
- More than 2.5 times the standard error are replaced entirely and are given a weight of 0, and
- Between 1.5 and 2.5 are partially replaced using a weight function that is a linear interpolation between 0 and 100 based on the value of the irregular.

The final weights for the irregular component are in Table C17. In the example below, you can see values where X-12 did find that the point was extreme. For example, May 1979 had a value greater than 2.5 times the standard error because that point has a weight of zero. March 1979 had a value slightly less than 2.5 times the standard error because the point has a weight of only 8. May and June of 1978 were also partially replaced.

By default, the cutoffs for the calculations are 1.5 and 2.5. You can see them listed as the "sigma limits" in the heading information in the table. You can change the sigma limits in the program with the argument sigmalim in the x11 spec. Higher values for the sigma limits would identify fewer extreme values.

For these tables, we are looking for any year or any month/quarter with a lot of extreme values, or any month/quarter with several extreme values in a row for a given month/quarter.

Example—Table C17

```
C 17  Final weights for irregular component
  From  1978.Jan to 2003.Apr
  Observations        304
  Lower sigma limit   1.50
  Upper sigma limit   2.50
 --------------------------------------------------------------------
            Jan     Feb     Mar     Apr     May     Jun
            Jul     Aug     Sep     Oct     Nov     Dec     S.D.
 --------------------------------------------------------------------
   1978    100.    100.    100.    100.     68.     48.
           100.    100.    100.    100.    100.    100.    13848.
   1979    100.    100.      8.    100.      0.    100.
           100.    100.    100.    100.    100.    100.    13848.
   1980    . . .
```

Tables D8 and D9 contain the Final Unmodified SI Ratios and the Final Replacement Values for the SI Ratios. The SI Ratio is the ratio of the original series to the estimated trend, so it is an estimate of the detrended series. It can also be thought of as the seasonal-irregular component. You will notice in the example below that the months that contain replacement values correspond to the months with weights less than 100 in the Table C17 above.

Example—Tables D8 and D9

```
D 8  Final unmodified SI ratios
  From  1978.Jan to 2003.Apr
  Observations        304
 --------------------------------------------------------------------
            Jan     Feb     Mar     Apr     May     Jun
            Jul     Aug     Sep     Oct     Nov     Dec     AVGE
 --------------------------------------------------------------------
   1978     99.2    98.9    99.1    99.1    99.2   101.7
           102.0   101.2    99.8   100.2   100.1    99.8   100.0
   1979     98.9    99.2    99.4    98.9    99.0   101.3
           101.9   101.1   100.0   100.3    99.8    99.8   100.0
   1980    . . .
```

```
D 9  Final replacement values for SI ratios
  From  1978.Jan to 2003.Apr
  Observations        304
 ----------------------------------------------------------------
            Jan      Feb      Mar      Apr      May      Jun
            Jul      Aug      Sep      Oct      Nov      Dec
 ----------------------------------------------------------------
   1978   ******** ******** ******** ********    99.3   101.4
          ******** ******** ******** ******** ******** ********
   1979   ******** ********    99.1 ********    99.5 ********
          ******** ******** ******** ******** ******** ********
   1980   . . .
```

If there are some months with many extreme values, one possible solution is to change the seasonal filters for a given month. It may also help to change the sigma limits.

Acknowledgements

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